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Pergamon
Energy 28 (2003) 769–788
ENERGY
www.elsevier.com/locate/energy

Energy in the theory of production

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Received 4 May 2002

Abstract

The fundamental role of energy as a factor of production is investigated. In this paper, capital K, labour L and work of production equipment—productive energy S—are considered to be important production factors. However, in contrast to some theories, the author does not consider the variables K, L, S to be independent; energy and labour inputs act as substitutes for each other, while capital K and work (L and S) are complements. Equations of production dynamics as a set of equations for variables (output Y, value of production equipment K, labour L, energy S, and two technological variables $\bar{\lambda}$ and $\bar{\varepsilon}$ connected with labour requirement and energy requirement, respectively) are established. The time path of output is determined by the exogenous quantities: the potential labour supply and the availability of energy resources. This theory is an extension of the conventional two-factor theory of economic growth. In the previous theory, capital played two distinctive roles which are separated in the present theory: capital as value of production equipment and capital as a substitute for labour. In the latter case, capital is the means by which the labour resource is substituted by energy rather than a production factor itself. Empirical evidence from the US economy is used to estimate the validity of the proposed mechanism of economic growth.

1. Introduction

One of the ‘eternal’ problems of the theory of production is how to explain the growth of output, i.e., Gross Domestic Product (GDP) for a nation. GDP is a monetary measure of material and spiritual achievements of a society for a year, and the problem is how to connect change of GDP with changes of some other variables [1]. The conventional approach to the problem, classically exposed in a famous work by Cobb and Douglas [2], is to consider output Y to be determined by production equipment measured by its value K (capital) and work of labourers L measured,

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doi:10.1016/S0360-5442(03)00031-8

for example, in working hours per year. However, it has long been argued [3–6] that energy is vital to the performance of the economy. Energy must be considered not only as an ordinary intermediate product that contributes to the value of produced products by adding its cost to the price, but also as a value-creating factor which has to be introduced[1] in the list of production factors in line with production factors of conventional neo-classical economics—capital K and labour L.

The relationship between energy consumption[2] and economic growth has become an especially dramatic issue. Some scholars consider energy to have nothing to do with the production of value, and the production function reduces to $Y = Y(K, L)$; others feel that energy is a crucial production factor. However, attempts to use econometric methods to elucidate the role of energy in the production of value have met with some major difficulties. Although one can easily estimate the total quantity of primary energy carriers consumed in production, the final consumption of energy is a subject of discussion. These problems, arising from attempts to estimate how much energy is usefully converted in production processes, are discussed by Patterson [11], Nakićenović et al. [12], Zarnikau et al. [13] and Ayres [14]. According to Nakićenović et al. [12], the world average of consumption efficiency (from primary to final use) was about 30% in 1990, while it was higher in developed countries. Data collected by Ayres [14, Tables 2–4] show that the efficiency of energy conversion increased during the last centuries, whereas his figures of efficiencies are much less: 0.00847 in 1800 compared with 0.04376 in 1979 in the US economy.

Apart from this, one has to take into account that energy participates in production processes in different ways: part of the energy (correctly: energy carriers) is consumed as a plain commodity. So, for example, 0.55 quads of oil products from the total amount of about 97 quads of primary energy[3] consumed in the US economy in the year 1999 was laid on the roads. Clearly, it is not energy content that is important here, but the property of oil products as specific materials. Energy is the most important intermediate product in the production of aluminium, metallurgical operations, and some chemical processes, among others. Energy can directly be used as a final product (space heating, lighting, and so on). In all these cases, the cost of energy is included in the cost of the final products, and energy carriers are quite similar to other intermediate products participating in production process. In other cases, apart from being a product, energy from external sources is used to substitute for labour in technological processes. Energy-driven equipment works in the place of manual labour and acquires all the properties of a production factor, including the property to produce surplus value. In these cases, energy is used as a value-creating production factor.

[1] The introduction of energy could be justified from a thermodynamic point of view [7]. One can consider the process of production of useful things to be a process of creating of far-from-equilibrium objects as explained by Prigogine [8,9]. To create and maintain these structures in our environment, as in any thermodynamic system, there is a need for matter and energy fluxes coming through the system [9,10]. Here, energy is coming in the form of human energy and energy from external sources that can be used with the help of proper equipment. The production system of society is a mechanism that attracts a huge amount of energy to transform matter into things that are useful for human beings. Production of useful things can be regarded as connected with the order (complexity) created in the environment by human activity.

[2] For the sake of precision the word *consumption* should be replaced by the word *conversion*. Energy cannot be *used up* in production process, but can only be converted into other forms: chemical energy into heat energy, heat energy into mechanical energy, mechanical energy into heat energy and so on. The measure of converted energy (work) is exergy.

[3] Primary energy is the name for the amount of primary energy carriers (oil, coal, running water, wind and so on) measured in energy units. It is convenient to measure huge amounts of energy in a special unit, *quad* (1 quad = 10^{15} Btu$\approx 10^{18}$ J), which is usually used by the US Department of Energy.

Thus, distinguishing between the two roles of energy in production processes is important. A part of consumed energy—let us call it *productive energy S*—has to be considered not only as an ordinary intermediate product that contributes to the value of produced products by adding its cost to the price, but also as a value-creating factor which has to be introduced in the list of production factors equally with production factors of conventional neo-classical economics—capital K and labour L. Production of value Y has to be determined by three production factors

$$Y = Y(K,L,S). \tag{1}$$

The argument S of the production function has to be interpreted as genuine work done by production equipment with the help of external sources of energy instead of labourers. This quantity can be considered as capital service provided by capital stock. One clearly has to know how to estimate the amount of energy that really determines production of value. This problem was recently reviewed and discussed by Cleveland et al. [15].

The proposed theory has been designed to consider the phenomenon of production of value. The theory begins with a description of a production system that may be viewed as a collection of equipment (measured by its value K), acquiring its ability to act from labour (L) and productive energy (S) inputs, while a method for separating productive energy from the total primary consumption of energy appears to be an organic part of the theory. The method of separation allows us to calculate the growth rate of productive energy. Dynamic equations for the production factors K, L and S and the technological variables, which are characteristics of the quality of production equipment, are set up. Eventually, one obtains a system of evolutionary equations which imitate the behaviour of the production system. In Section 2, the paper discusses the main principles of the theory, whereas details of some arguments can be found elsewhere [7].[4] In Section 3, the ability of the theory to describe a real situation is illustrated for the US economy.

2. Derivation of equations of growth

2.1. Technology and dynamics of production factors

From a material point of view, the process of production is a process of transformation of raw materials into finished and semi-finished items, semi-finished items into other semi-finished and finished items and so on, until the finished items, which can be used by man, are made. We can observe how clay is transformed into pots, how clay, sand, and stone are transformed into buildings, how ores and raw materials are transformed into an aeroplane. The applied technology determines, first, what one needs to produce, and determines the material side of the process of production. Different appliances have been invented to do transformations. This is a material realisation of technology, i.e., production equipment. The entire collection of it is estimated by its value K, which obeys the balance relation [16]

[4] The principles of the theory were discussed in the monograph [7], though some issues have to be reformulated. In particular, the concept of productive energy was not clearly defined, and the important contribution to production of value from structural change was erroneously omitted.

$$\frac{dK}{dt} = I - \mu K, \tag{2}$$

where production investment I is a part of output which is accumulated in the material form of production equipment, while the other part is designed for consumption. The second term on the right side of Eq. (2) describes the decrease of capital due to removal from service with the depreciation coefficient μ. The amount and the distribution of production capital with time are thus comprehensively determined by the history of investments. The major part of production capital comes from recent investments.

From the other side, to produce something involves doing some work. The work can either be done by a labourer, or by some external energy sources which can be used to do the same work. So, for example, to grind corn into flour a man can use stones, or a hand mill, or a water mill, or a steam mill. The same results can be obtained with different consumed energy and with different labourer's work. No matter who or what does the work: the whole work must be done to obtain the result. The technology determines how much work of human beings L and work of external sources (wind, water, coal, oil, ...) S is necessary for production, while human work can be substituted by work by external sources of energy. Although one needs production equipment (capital stock) to attract an extra amount of energy to substitute for labour, work (labour services) can be replaced only by work (capital services), not by capital stock.[5] The capital is an intermediate agent to attract energy.

The expansion of production, characterised by changes of the accumulated value, requires additional labour and energy, so that dynamics of the production factors can be derived as the balance equations

$$\frac{dL}{dt} = \lambda I - \mu L, \qquad \frac{dS}{dt} = \varepsilon I - \mu S. \tag{3}$$

The first terms in the right side of these relations describe the increase in the quantities caused by gross investments I; the second terms reflect the decrease in the corresponding quantities due to the removal of a part of the production equipment from service. Coefficients λ and ϵ determine the required amount of labour and external energy per unit of investment and are characteristics of introduced technology: therefore, they can be denominated as technological coefficients: the labour requirement (λ) and the energy requirement (ϵ). It is convenient to use the dimensionless technological variables

$$\bar{\lambda}(t) = \frac{K}{L}\lambda, \qquad \bar{\varepsilon}(t) = \frac{K}{S}\varepsilon.$$

If these quantities are less than unity, it means that labour-saving and energy-saving technologies are being introduced at the time.

From Eqs (2) and (3) the technological variables can be written as

[5] As is formulated by Odum [17, p. 263]: "To truly substitute for something in a production process means to find an alternative with equivalent transformity and emergy contribution."

$$\bar{\lambda} = \frac{\nu + \mu}{\delta + \mu}, \qquad \bar{\varepsilon} = \frac{\eta + \mu}{\delta + \mu}, \tag{4}$$

where notations for the growth rates of production factors are introduced:

$$\delta = \frac{1}{K}\frac{dK}{dt}, \qquad \nu = \frac{1}{L}\frac{dL}{dt}, \qquad \eta = \frac{1}{S}\frac{dS}{dt}. \tag{5}$$

The depreciation coefficient μ can be excluded from Eqs (4). So, one can obtain the relation between different real rates of growth

$$\delta = \nu + \alpha(\eta - \nu), \qquad \alpha = \frac{1-\bar{\lambda}}{\bar{\varepsilon}-\bar{\lambda}}. \tag{6}$$

The introduced quantity—the technological index α—appears to be a very important characteristic of the production system. Eq. (6) is a useful relation between the unknown growth rate of productive energy η and the technological index α, considering the two growth rates (δ and ν) to be known.

Let us note that the simple relations in this subsection were obtained with the simplifying assumption that deterioration of labour and productive energy coincides with deterioration of capital stock, that is, the coefficients of depreciation in Eq. (3) are equal to the one in Eq. (2). Considering a more general case is not difficult.

2.2. Investment and dynamics of the technological coefficients

To find an expression for investment I, let us consider demand and supply of production factors in terms of rates of growth. Demand of production factors is determined by Eqs (2) and (3), so the rates of real growth of the production factors are defined by the following relations:

$$I - \mu K = \delta K, \qquad \lambda I - \mu L = \nu L, \qquad \varepsilon I - \mu S = \eta S. \tag{7}$$

To characterise an offer of the production factors, we introduce the rates of potential growth $\tilde{\delta}(t)$, $\tilde{\nu}(t)$ and $\tilde{\eta}(t)$, while the rate of capital potential growth $\tilde{\delta}(t)$ is reasonable to be determined, similar to Eq. (6), by the relation

$$\tilde{\delta} = \tilde{\nu} + \alpha\ (\tilde{\eta} - \tilde{\nu}). \tag{8}$$

The growth in the labour offer $\tilde{\nu}(t)$ is determined by the population size and, in addition, depends on the cost of labour. The rate of potential growth of energy $\tilde{\eta}(t)$ is a more uncertain quantity. To use external energy in production, some devices have to be invented, made and installed for work. One ought to have available sources of energy and appliances which use energy for production. So, the supply of energy is determined by fundamental results of science, by research, by project works, and by materialisation of all human imagination about how to use energy for production. The base for the use of energy is a stock of knowledge, that is, a mass of suspended, deposited messages which are useless unless they are materialised in routine production processes. This determines the possibility for society to attract extra energy to production and, eventually, the quantity $\tilde{\eta}(t)$. One can find plenty of brilliant examples of ‘transformation’ of knowledge into useful energy in the history of technology. However, little is known about formal description of this process, so that one can only guess that energy supply, that is, how much energy can be used by society, is determined by available knowledge. From this point of view, one can consider stock of knowledge as a genuine source of economic growth.

Anyway, the rates of real growth of production factors do not exceed the rates of potential growth, so that one can write the relations

$$\delta \leq \tilde{\delta}, \quad \nu \leq \tilde{\nu}, \quad \eta \leq \tilde{\eta},$$

which, taking relations (7) into account, implies

$$I \leq (\mu + \tilde{\delta})K, \qquad I \leq \frac{\mu + \tilde{\nu}}{\lambda}L, \qquad I \leq \frac{\mu + \tilde{\eta}}{\varepsilon}S. \tag{9}$$

If no more restrictions are imposed, investment in the production sector is

$$I = \min\begin{cases}(\tilde{\delta} + \mu)K \\ (\tilde{\nu} + \mu)K/\bar{\lambda}. \\ (\tilde{\eta} + \mu)K/\bar{\varepsilon}\end{cases} \tag{10}$$

We have, thus, obtained three modes of economic development according to the three opportunities in formula (10). The first one is valid for the case of abundance of labour, energy, and raw materials. Internal restrictions are imposed on development of the system here. The second one is valid for the case of lack of labour and abundance of energy and raw materials. The third one is valid for the case of lack of energy and abundance of labour and raw materials. In the last two of the three modes, one of the external production factors is limited. The conditions for the realisation of each of the three modes can be written as

$$\begin{matrix} \tilde{\nu} + \mu \geq (\tilde{\delta} + \mu)\bar{\lambda} & (\tilde{\nu} + \mu)\bar{\varepsilon} \leq (\tilde{\eta} + \mu)\bar{\lambda} & (\tilde{\eta} + \mu)\bar{\lambda} \leq (\tilde{\nu} + \mu)\bar{\varepsilon} \\ \tilde{\eta} + \mu \geq (\tilde{\delta} + \mu)\bar{\varepsilon} & \tilde{\nu} + \mu \leq (\tilde{\delta} + \mu)\bar{\lambda} & \tilde{\eta} + \mu \leq (\tilde{\delta} + \mu)\bar{\varepsilon} \end{matrix}. \tag{11}$$

The expansion of production system is related to the possibility to engage additional amounts of production factors. One can assume that the intrinsic property of the economic system is a trend to use all available resources. One can consider it as a principle of evolution of the production system. In fact, it is the principle of maximum power ([17], p. 20) applied to the production system. It means that the technological coefficients have tendencies to change in such a way that conditions (11) are being relaxed. The change is connected with internal processes of developing and propagation of known technologies. In the first approximation, these tendencies of technological changes are described by equations for the dimensionless technological variables

$$\frac{d\bar{\lambda}}{dt} = -\frac{1}{\tau}\left(\bar{\lambda} - \frac{\tilde{\nu} + \mu}{\tilde{\delta} + \mu}\right), \qquad \frac{d\bar{\varepsilon}}{dt} = -\frac{1}{\tau}\left(\bar{\varepsilon} - \frac{\tilde{\eta} + \mu}{\tilde{\delta} + \mu}\right), \tag{12}$$

where τ is the time of introduction of production equipment into action, that is, time of crossover from one technological situation to another.

Now one can directly calculate the change of the technological index α, defined by Eq. (6), during the evolution of the system. Eqs (12) determine the derivative of the technological index

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{\tilde{\delta}-\tilde{\nu}-\alpha(\tilde{\eta}-\tilde{\nu})}{\tau(\bar{\varepsilon}-\bar{\lambda})(\tilde{\delta}+\mu)}. \tag{13}$$

One can see that, if relation (8) is valid, the technological index appears to be an integral of evolution and, therefore, can be considered as a very important characteristic of the production system.

2.3. Production of value and the productivity principle

To determine production of value, one needs to reduce function (1) to the form which is consistent with the above technological description of the production process.[6] First, as far as there is a relation (6) among the growth rates of the production factors, the variables K, L and S might be interdependent: only two of the arguments of function (1) are independent.[7] Then, the technological description assumes that one ought to consider energy and labour inputs as substitutes for each other, and amount of production equipment, universally measured by its value K, has to be considered a complement to work (L and S) of the production equipment.[8] All this urges us to write the production function (1) in the form of the two alternative lines

$$Y = \begin{cases} Y(K) \\ Y(L,S) \end{cases}, \qquad \mathrm{d}Y-\Delta\mathrm{d}t = \begin{cases} \xi(K)\,\mathrm{d}K \\ \beta(L,S)\,\mathrm{d}L + \gamma(L,S)\,\mathrm{d}S \end{cases} \tag{14}$$

where $\Delta\,\mathrm{d}t$ is a part of an increment of production of value which is connected with change of characteristics of the production system (the structural change). In line with the existing economic theories, the quantities ξ, β and γ can be labelled as marginal productivities of the corresponding production factors. Considering that the structural change is zero, the marginal productivity ξ corresponds to value produced by addition of a unit of capital; the marginal productivities β and γ correspond to value produced by addition of a unit of labour input at constant external energy consumption and by the addition of a unit of energy at constant labour input, respectively. We have to consider that all marginal productivities are positive. One uses production factors to create useful commodities and an addition of any production factor must increase in production of things—this is known as the productivity principle.

One can use equations for the production factors (2) and (3) to rewrite relations (14) for production of value in the form

$$\frac{\mathrm{d}Y}{\mathrm{d}t}-\Delta = \begin{cases} \xi(I-\mu K) \\ (\beta\lambda + \gamma\varepsilon)I-\mu(\beta L + \gamma S) \end{cases}. \tag{15}$$

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[6] The relationship between market value Y and production factors itself is an amazing relation. One can state that production process determines the production cost of products, but why does it correlate with the market evaluation? To continue, we share a belief in the existence of such correlation.

[7] One can see that Eq. (28) is an inexplicit relation among the production factors.

[8] The complementarity of energy and capital was analysed by Berndt and Wood [6]. They remark on p. 351 "... that *E–K* complementarity and *E–L* substitutability are consistent with the recent high-employment, low-investment recovery path of the US economy." Patterson [11, p.382] found "for New Zealand (1960–1985) that energy and labour inputs acted as mild substitutes to each other, and energy and capital inputs were mild complements to each other". The researches dealt with the total primary consumption of energy E and could not discover the relationship of exact substitution between labour L and productive energy S.

The right-hand sides of these equations are equal to each other, so that one can write equations for the characteristic parameters of the system and discover relations among the marginal productivities

$$\beta = \xi\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}-\bar{\lambda}}\frac{K}{L}, \qquad \gamma = \xi\frac{1-\bar{\lambda}}{\bar{\varepsilon}-\bar{\lambda}}\frac{K}{S}. \tag{16}$$

If the technological coefficients $\bar{\lambda}$ and $\bar{\varepsilon}$ take arbitrary values, Eqs (16) permit one of the marginal productivities, but not both, to be negative. One can see that, if relations

$$\bar{\lambda} < 1 < \bar{\varepsilon} \quad \text{or} \quad \bar{\lambda} > 1 > \bar{\varepsilon} \tag{17}$$

are valid, the marginal productivities are non-negative, so that these relations can be considered as an expression of the productivity principle.

The forms (14) are consistent with some different approaches to the theory of production of value [17,18]. The present theory keeps the main attributes of the neo-classical approach, i.e., the concept of value produced by production factors (donor value) and concept of production factors themselves, and can be considered as a generalisation and extension of the conventional neo-classical approach, while the roles of production factors are revised. In the conventional, neo-classical theory, capital as variable played two distinctive roles: capital stock as value of production equipment and capital service as a substitute for labour. We consider capital stock to be the means of attracting labour and energy services to production, while human work and work of external energy sources are considered the true sources of value.[9] Human work is replaced by work of external energy sources by means of different sophisticated appliances. In contrast with the conventional theory, the perfect substitution of labour and energy does not lead to any discrepancies. One can imagine a factory working without energy or without labour, but, of course, one cannot imagine a factory without production equipment.

Now, one can write a simple approximation for the marginal productivities. So that the description ought to be valid for any initial point, one approximates production function, assuming also that production is homogeneous, as a power function

$$Y = Y_0\frac{L}{L_0}\left(\frac{L_0 S}{L S_0}\right)^{\alpha}, \tag{18}$$

where α is a characteristic of the production system which, as shown below, coincides with the technological index introduced by Eq. (6). The above relations provide the following expressions for marginal productivities and the contribution from structural change

$$\beta = Y_0\frac{1-\alpha}{L_0}\left(\frac{L_0 S}{L S_0}\right)^{\alpha}, \qquad \gamma = Y_0\frac{\alpha}{S_0}\left(\frac{L_0 S}{L S_0}\right)^{\alpha-1}, \qquad \Delta = Y\ln\left(\frac{L_0 S}{L S_0}\right)\frac{d\alpha}{dt}, \tag{19}$$

[9] One can argue that, here, labour can be reduced to energy, and one has the only argument energy as a source of value. However, labour and energy are measured in different units and nobody knows how to calculate real work provided by these production factors and compare them. Besides, if possible, such comparison could not be universal, so that dealing with the two separate arguments is better.

where L_0 and S_0 are values of labour and capital services in the base year. Having compared expressions (16) and (19) for marginal productivities, one obtains

$$\xi = Y_0 \frac{L}{L_0 K} \left(\frac{L_0 S}{L S_0} \right)^{\alpha}, \qquad \alpha = \frac{1-\bar{\lambda}}{\bar{\varepsilon}-\bar{\lambda}}. \tag{20}$$

Thus, the index α in Eq. (18) is the same quantity as introduced by Eq. (6). The productivity principle restricts values of the technological index, $0 < \alpha < 1$. Besides, all available information about the technological performance could be introduced by estimating this quantity. Moreover, a condition regarding the optimal use of production factors enables us to establish a relation between the parameter α on the one hand and the shared costs of production factors on the other. This provides the different means of estimating the technological index.

3. Application to the US economy

The main result of the previous section can be formulated as a system of equations for the dynamics of a production system, that is, Eqs (2), (3), (10), (12), (14) and (19), which represent a closed set of equations for six variables: gross domestic product Y, value of production equipment K, labour L, productive energy S, and two technological variables, namely, the labour requirement $\bar{\lambda}$ and the energy requirement $\bar{\varepsilon}$. The technological variables combine to create the technological index α—an assumed integral of evolution. The system contains two internal characteristics: the rate of depreciation μ and the time of technological rearrangement τ. These quantities have to be given, as one cannot consider them as arbitrary ones. The rates of potential growth of labour and energy $\tilde{v}(t)$ and $\tilde{\eta}(t)$ ought to be given as exogenous functions of time. So, the written system of equations allows us to draw scenarios of evolution of national economies for possible development of available production factors. As the equations are evolutionary ones, initial values of all variables have to be given. To illustrate the applicability of the system of equations to describe a real situation, we refer to governmental statistical data for the US economy for years 1890–1999 (see Appendix A). To find trajectories of evolution of variables due to the evolution equations, the simplest numerical method (Euler's method, the details of calculations are omitted) was used.

3.1. The technological index and personal consumption

The technological index α is a very important characteristic of the production system. The value of the technological index whereas one cannot consider them regarding the optimal use of production factors. One can assume that production factors L and S are chosen in such amounts, so that they must be the most effective in production, that is, values of production factors maximise production function (18) at given total expenses for production factors

$$cL + pS = V,$$

where c and p are cost of 'consumption' of the production factors, and V is a part of gross output which goes towards maintenance of production factors.

One can follow the conventional method of finding a conditional extremum to sort out the extremum point and to find that the technological index α can be expressed through prices and amounts of production factors

$$\alpha = \frac{pS}{cL + pS}. \tag{21}$$

So, the technological index α represents the share of expenses needed for utilisation of capital services in total expenses for production factors. If production factors are chosen as optimal, then

$$0 < \alpha < 1,$$

which coincides with conditions of the positivity of marginal productivities (see formulae (17) in the previous section).

Expression (21) allows one to estimate the technological index α due to estimates of cost of consumption of production factors. Clearly, the amount of value which is needed to support capital services S during a year is equal to μK, so that the cost of consumption of capital services is

$$p = \frac{\mu K}{S}. \tag{22}$$

The current consumption $C = cL$ is defined as the value of the minimum amount of products which are needed for the humans to subsist. Perhaps the proper quantity to characterise the necessary consumption is the poverty threshold used in the US statistics. The estimates of these quantities for a person in different family situations since year 1959 can be found on the US Bureau of Census website (http://www.census.gov). One can consider the poverty threshold for a single person to be a realistic estimate of the current consumption. For year 1996, for example, this quantity is estimated as US$ 7995 per person per year. This quantity ought to be multiplied by the number of persons to get an estimate of the consumption in year 1996 as C = US$ 2120 billion compared with the expenses for maintenance of consumption of capital services $pS = \mu K$ = US$ 1378 billion (1996). So, for the last decade of the 20th century one can get $\alpha \approx 0.4$. It means that about 40% of total expenses for production factors take energy as substitute for labour.

3.2. Production factors and the technological index

Empirical values of capital K, labour L and total primary consumption of energy E are known and depicted on the plot in Fig. 1 with solid lines. The third production factor—productive energy S—has to be estimated. We use a simple method which allows us to calculate both productive energy S and values of the technological index α.

The value of the technological index α can be calculated, from Eq. (18) as

$$\alpha = \frac{\ln\left(\frac{Y L_0}{Y_0 L}\right)}{\ln\left(\frac{L_0 S}{L S_0}\right)}. \tag{23}$$

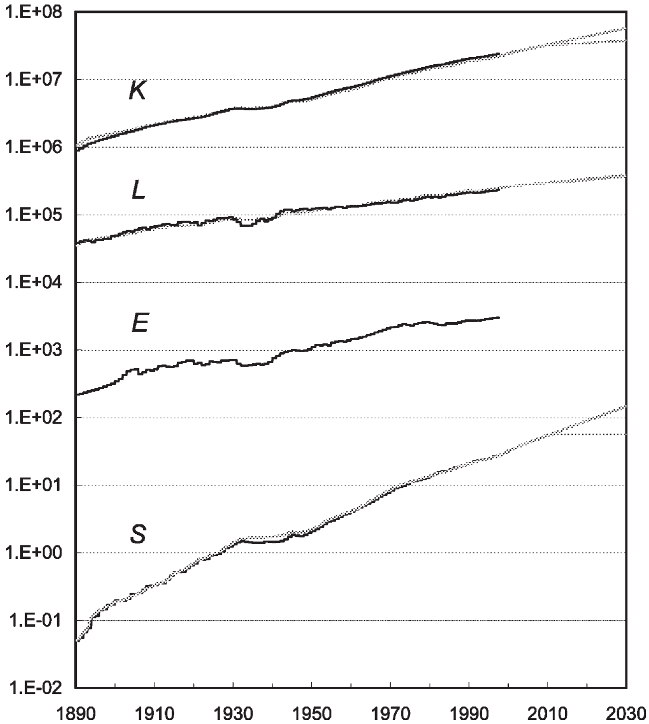

Fig. 1. Production factors in the US economy. Basic production equipment (capital stock) K in million 1996 dollars; consumption of labour L in million working hours per year; total and productive consumption of energy E and S, correspondingly, in quads per year. The solid lines represent empirical values, while the dotted lines show the results of the calculation.

However, amount of productive energy S itself depends on the value of the technological index α. The rate of growth of productive energy, from Eq. (6), is calculated as

$$\eta = \frac{\delta - (1-\alpha)\nu}{\alpha}, \quad 0 < \alpha < 1. \tag{24}$$

Then, the time dependence of productive energy can be restored by solving the equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \eta(\alpha)S. \tag{25}$$

Eqs (23)–(25) allow one to estimate the technological index α and productive energy S at given time series of Y, K and L.

The results of calculation for α are depicted on the plot of Fig. 2 in line with the values of α calculated from available data of the US Bureau of Census for the poverty threshold which is taken as personal consumption. Note that the choice of initial value of the technological index allows us to move the whole curve of α up and down, so that it is important to have at least one point where the absolute value of α is known, which, according to a previous estimate, is taken as $\alpha \approx 0.4$ in year 1996. The values of the technological index can be considered approximately

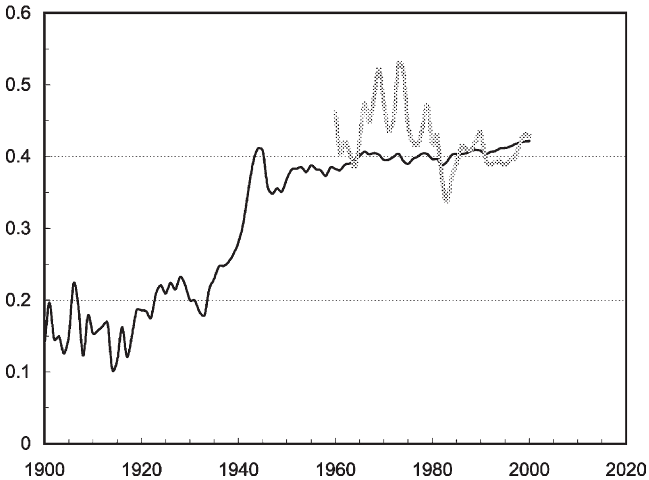

Fig. 2. Technological Index. Solid line represents values of α found according to Eqs (23)–(25). The dotted line represents values calculated from the values of the poverty threshold.

constant in years 1950–1999, that is, there are some periods when the technological index α can be indeed considered as an approximate integral of evolution.

The values of the technological index allow us to calculate the growth rate of productive energy η and to restore the time dependence of the quantity. The results for S are shown in Fig. 1 as a solid curve in line with the other production factors. One can see that the productive energy is growing on average faster than the total energy in years 1900–2000; however, there are some years of recession. Note that, to reproduce the time dependence of productive energy, one must have at least one point where absolute value of productive energy is known. One can consider data for consumption of energy by vehicles of transportation, machine-tools of manufacturing and appliances of information technology, which, supposedly, substitute for corresponding human efforts, to estimate the amount of productive energy in the US economy as 1 quad for the year 1999. It is approximately 100 times less than total (primary) consumption of energy which was about 97 quads in 1999. However, the amount of primary energy (energy carriers), which is needed to provide this amount of productive energy, is about 25 quads. It is about 26% of total primary consumption of energy. This estimate corresponds with estimates by Ayres ([14], Table 1), who found that the part of energy used for machine drive, transport drive, farming and construction in the US economy was 9% in year 1800, 23% in 1900 and about 32% in 1991.

One can see that the value of the technological index for years 1950–2000 can be considered approximately constant $\alpha \approx 0.4$, and the time dependencies of the production factors for these years can be approximated by the exponential functions

$$K = K_0\, e^{\delta t}, \qquad L = L_0\, e^{\nu t}, \qquad S = S_0\, e^{\eta t}, \tag{26}$$

where $\delta = 0.0316$, $\nu = 0.0146$, $\eta = 0.0588$. It is not surprising that the growth rate of productive energy is different from the one of primary energy. Note that it is closer to the growth rate of consumption of electricity, so that one can assume that electric energy was one of the main contributors to productive energy in the US economy in the 20th century.

To illustrate the applicability of the theory, trajectories of growth of the production factors can be calculated from Eqs (2), (3), (10), (12) and (13), that is

$$\frac{\mathrm{d}K}{\mathrm{d}t} = I-\mu K, \qquad \frac{\mathrm{d}L}{\mathrm{d}t} = \left(\bar{\lambda}\frac{I}{K}-\mu\right)L, \qquad \frac{\mathrm{d}S}{\mathrm{d}t} = \left(\bar{\varepsilon}\frac{I}{K}-\mu\right)S,$$
$$\frac{I}{K} = \min\left\{(\tilde{\delta}+\mu),(\tilde{\nu}+\mu)\frac{1}{\bar{\lambda}},(\tilde{\eta}+\mu)\frac{1}{\bar{\varepsilon}}\right\}, \qquad \frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{\tilde{\delta}-\tilde{\nu}-\alpha(\tilde{\eta}-\tilde{\nu})\mathrm{d}\bar{\lambda}}{\tau(\bar{\varepsilon}-\bar{\lambda})(\tilde{\delta}+\mu)\mathrm{d}t} = -\frac{1}{\tau}\left(\bar{\lambda}-\frac{\tilde{\nu}+\mu}{\tilde{\delta}+\mu}\right), \tag{27}$$
$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}t} = -\frac{1}{\tau}\left(\bar{\varepsilon}-\frac{\tilde{\eta}+\mu}{\tilde{\delta}+\mu}\right), \qquad \alpha = \frac{1-\bar{\lambda}}{\bar{\varepsilon}-\bar{\lambda}}.$$

Scenarios of development can be obtained, if one sets the rates of potential growth of capital, labour and productive energy $\tilde{\delta}$, $\tilde{\nu}$ and $\tilde{\eta}$. We chose to use empirical values of the technological index α and the rates of potential growth of labour and productive energy as exogenous quantities. Before the year 1999, the rates of potential growth of labour and productive energy $\tilde{\nu}$ and $\tilde{\eta}$ are taken to be a little bit more than the rates of real growth to reproduce the empirical dependencies of L and S. Beyond the year 2000, we explore two scenarios of development. In both cases, the rate of growth of labour $\tilde{\nu}$ coincides with the rate of population growth, namely, $\tilde{\nu} = 0.01$ for the US. The first scenario corresponds to value $\tilde{\eta} = 0.05$ for all years. The second one shows the effect of diminishing the energy supply in the economy: the value of $\tilde{\eta} = 0.05$ in year 1999 decreases to zero in year 2010. The dotted lines in Fig. 1 show the results of calculation of the production factors at values of the depreciation coefficient μ calculated according to cited statistical data ($\mu \approx 0.02$ before year 1925 and increases from 0.026–0.068 over years 1925–1999) and time of technological rearrangement $\tau = 1$ year. The initial values of all variables, apart from the technological variables, are known from empirical data. The initial values of the technological variables can be chosen arbitrarily, because, due to the relaxation equations from set (27), the initial values of the technological variables are being forgotten in $\tau = 1$ year. However, the choice of the technological variables must correspond to the value of the technological index α.

3.3. Marginal productivities

The previous results allow one to calculate the contribution from the structural change Δ and one can estimate the marginal productivities ξ, β and γ from differential formulae (14) and empirical data. The marginal productivities are connected to each other by Eqs (16) which are followed by another simple relation for the marginal productivities

$$\xi = \beta\frac{L}{K} + \gamma\frac{S}{K}. \tag{28}$$

One has a unique opportunity to confirm the consistency of the proposed description by testing relation (28). The results of calculation of the left and right sides of Eq. (28) for the US economy are depicted in Fig. 3. For the reliable years 1950–2000, the average value of the capital-stock marginal productivity is (0.309 ± 0.035) year^{-1}, whereas the average value of the right-hand side of Eq. (28) is (0.320 ± 0.041) year^{-1}. The values of the marginal productivity practically coincide with the averaged bulk productivity Y/K, which is (0.318 ± 0.010) year^{-1}; this is evidence that

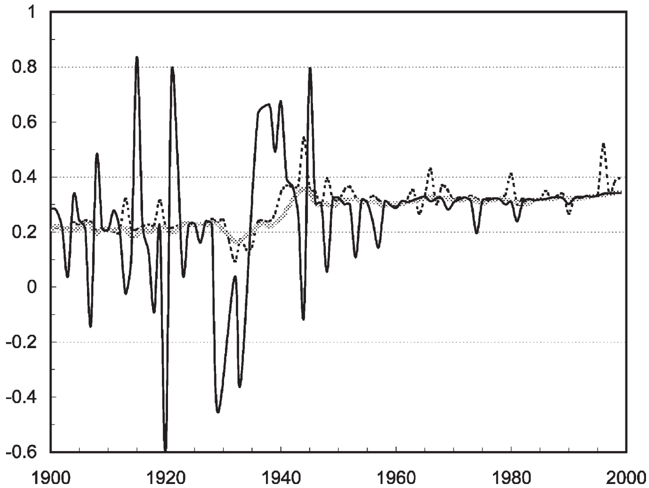

Fig. 3. The marginal productivity of capital in the US economy. The solid line represents direct estimates of ξ from the empirical data and the equation $dY-\Delta\, dt = \xi\, dK$. The dashed line shows the marginal productivity calculated according to Eq. (27), while β and γ are estimated directly from the empirical data and the equation $dY-\Delta\, dt = \beta\, dL + \gamma\, dS$. The dotted line represents the ratio Y/K.

the capital marginal productivity does not depend on argument K. In virtue of the dynamic equations for the production factors, ξ = const at α = const, and a change of the capital marginal productivity ξ during time is connected with a change of the index α.

Thus, the marginal productivity of capital stock can be considered as the ‘sum’ of the marginal productivities of labour and productive energy and no other factors are needed to include in the production function. Productivity of capital stock is, in fact, productivity of labour and energy, and the main result of technological progress is the substitution of human energy by energy from external sources by means of different sophisticated appliances. The production system of society is a mechanism which attracts a huge amount of energy to transform matter into things that are useful for human beings.

3.4. Production of value

Empirical values of GDP for the US economy for years 1890–1999 (see Appendix A) are depicted on the plot in Fig. 4 with a solid line. Production function (18), that is, the Cobb–Douglas production function

$$Y = Y_0 \frac{L}{L_0}\left(\frac{L_0 S}{L S_0}\right)^{\alpha}, \tag{29}$$

in which productive energy S stands in the place of capital stock K, allows one to calculate time dependence of output. At given time dependence of labour services L and at calculated values of the technological index α and productive energy S, time dependence of output identically coincides with the empirical one.

However, there is some interest, keeping in mind to test a possible method of forecasting, in

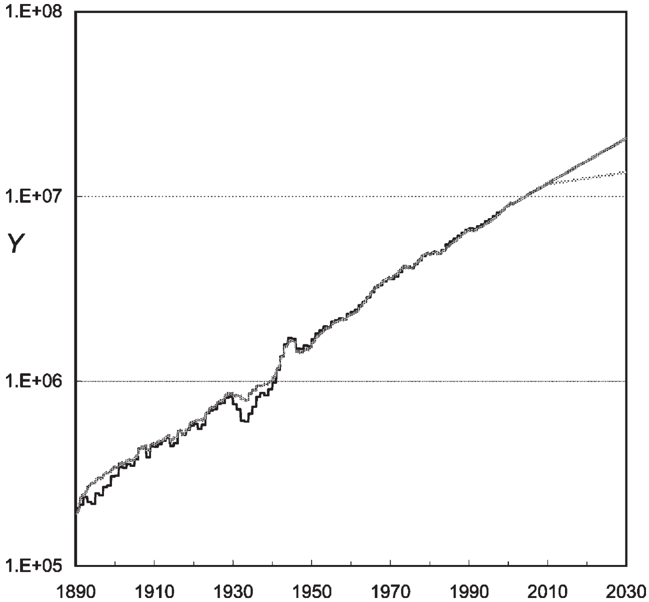

Fig. 4. Production of value in the US economy. Empirical (solid line) and calculated values of GDP in million 1996 dollars. At empirical values of production factors (see Fig. 1) and the technological index (see Fig. 2) the calculated and empirical curves are identical. The dotted lines show the result of the approximation: curves are depicted for empirical values of α and calculated values of production factors. Two scenarios are shown, while the lower line after year 2000 corresponds to diminishing supply of energy.

calculating an approximate trajectory of development at the values of the production factors that are solutions of Eqs (27). The dotted lines in Fig. 4 show the calculated time dependence of output as compared with an empirical amount of GDP. One can see that the calculated trajectory approximates the real time dependence of GDP, though we cannot correctly describe the output behaviour in the turmoil years 1930–1940. One can assume that a smooth development (at $\alpha =$ 0.4) will continue beyond year 2000. Outputs of two scenarios of development of the US economy for years 1950–2040, which correspond to the growth rates of labour and energy $\tilde{\nu}$ and $\tilde{\eta}$ as described above, are presented in Fig. 4. One can see a decrease in the growth rate of output in the case when the growth rate of productive consumption of energy is decreasing. Of course, these results ought to be considered an illustration of the method of forecasting rather than forecast itself. One needs to know the future availability of labour and productive energy to do a real prediction. However, it is interesting from this point of view to look at the consistency of the future estimations of growth rates of energy and GDP in the Annual Energy Outlook 2002 (ASO2002) with Projections to 2020 of the US Department of Energy (http://www.eia.doe.gov/oiaf/aeo/index.html). They assume the growth rates of total consumption of energy and GDP as 1.4 and 3%, respectively. However, to secure the assumed growth of GDP, one needs about 4% growth of the productive part of energy, which it is assumed can be reached by an increase in efficiency of final use of energy.

Note that, according to the previous results, the technological index for the relatively calm period of years 1950–2000 can be considered constant: $\alpha = 0.4$. There is some interest in describing the ‘stylised’ facts of economic growth, that is, exponential growth of output and production factors. Taking Eqs (26) into account, the output can be written in the following form:

$$Y = Y_0 \, e^{[\nu+\alpha(\eta-\nu)]t} = Y_0 \, e^{\delta t}. \quad (30)$$

One can see that the theory describes the ‘stylised’ facts of economic growth, while the growth rate of output is equal to the growth rate of capital and is connected with the growth rates of labour and energy. The empirical averaged growth rate of output 0.0326 is approximately equal to the growth rate of capital $\delta = 0.0316$. The contributions to the growth of output are $(1-\alpha)\nu \approx 0.0112$ from the labour growth and $\alpha\eta \approx 0.0235$ from the energy growth on average. Though capital is the means of attracting the production factors to production, increase in consumption of the production factors is connected with the increase in capital. One can separate the growth rate of capital δ in the growth rate of productive energy η to get the breakdown of the growth rate of output in conventional terms: the contribution from the labour growth $(1-\alpha)\nu \approx 0.0112$ and the contribution from the capital growth $\alpha\delta \approx 0.0126$. One can see that the Solow residual (total factor productivity) in this simple case of exponential growth can be expressed through the technological index and the growth rates as

$$\text{Solow residual} = \alpha(\eta-\delta) = (1-\alpha)(\delta-\nu) \approx 0.0109. \quad (31)$$

In the general case, the Solow residual includes also a contribution from the structural change.

4. Conclusion

The simplest schematisation of the production process allows us to formulate the simplest theory including only three production factors: capital stock *K*, labour services *L* and productive energy *S*. The first two are conventional, the third one is needed to explain the growth of productivity. In conventional terms, productive energy—work of production equipment—can be considered as capital service provided by capital stock, so that the developed theory of production keeps the main concepts of the conventional theory of production. We believe that the new formulation of the theory has some advantages, because it reveals the mechanism of growth by referring to the mechanism of the utilisation of external sources of energy and allows us, if the words of Solow [19] can be used, “*to model the endogenous component of technological progress as an integral part of the theory of economic growth*”.

Our investigation shows that the principle by which the evolution of the production system is followed is the principle of maximum power. It means that the production system is trying to devour all available resources. Energy can be considered a driving force of production; anyway, there is a strong correlation between output, from one side, and production consumption of labour and energy, from the other side. During the evolution, human work is being replaced by the work of different sophisticated appliances with the help of external energy sources. This is the main content of scientific and technological progress which is incorporated in the pattern of description quite naturally.

Acknowledgements

The author is grateful to the anonymous referees of the previous versions of the paper for detailed analysis and helpful comments. I am indebted to Professor Edward Mallia of the University of Malta who has kindly read through the manuscript and made many valuable suggestions for improving the English of the paper.

Appendix A. Data on the US Economy

Values of gross national product Y, gross investment I and capital K are available on a website of the US Bureau of Economic Analysis (http://www.bea.doc.gov). The values for Y from year 1959 and for I and K from year 1925 are reproduced in the table, while investment I is understood, in terms of the US Bureau of Economic Analysis, as a sum of investments in private fixed assets, in government fixed assets and in consumer durable goods which make up capital K. The time series for labour L for the latest decades (from year 1948) are found on a website of the US Bureau of Labour Statistics (http://www.stats.bls.gov). The series of relative quantities compiled by Scott [20] are used to restore absolute values of quantities for earlier years. Data for total consumption of energy E are taken from a website of the US Department of Energy (http://tonto.eia.doe.gov) for years from 1949 and from historical statistics [21] for earlier years. There is no need to discuss the discrepancies between data from different sources here, for we use the series not for analysis of economic growth but only for illustration of methods of analysis.

Year	GNP, $Y \times 10^{-6}$ \$ (1996)	Investment, $I \times 10^{-6}$ \$ (1996)	Capital, $K \times 10^{-6}$ \$ (1996)	Labour, $L \times 10^{-6}$ man-hour	Energy, E quad
1896	241,260	44,133	1,287,255	42,398	8.62
1897	267,115	53,596	1,331,494	44,119	9.02
1898	272,514	50,050	1,382,140	44,483	9.42
1899	305,550	62,951	1,432,732	48,624	9.91
1900	308,472	71,130	1,489,105	49,404	10.70
1901	343,044	79,186	1,543,855	52,181	11.69
1902	340,172	82,638	1,601,086	55,141	13.38
1903	355,526	81,633	1,666,279	57,160	15.06
1904	348,691	67,877	1,716,012	56,213	16.06
1905	378,409	73,291	1,774,784	59,734	16.35
1906	431,208	98,273	1,858,641	62,715	13.78
1907	434,922	96,940	1,947,528	64,516	14.67
1908	387,770	60,558	2,018,638	61,265	16.06
1909	443,541	96,205	2,071,670	65,471	15.46
1910	442,748	95,114	2,137,790	67,694	16.35
1911	457,558	83,589	2,199,302	69,182	17.94
1912	476,230	98,175	2,251,571	71,857	18.43

1913	495,300	109,430	2,326,498	72,900	17.74
1914	446,562	58,553	2,398,262	71,077	17.54
1915	460,975	56,161	2,445,923	70,822	17.94
1916	537,895	94,979	2,494,075	77,346	20.12
1917	513,626	75,380	2,548,021	79,096	20.91
1918	543,839	69,314	2,596,922	78,440	21.70
1919	575,043	116,507	2,643,806	75,961	21.70
1920	587,921	157,481	2,707,472	77,128	19.62
1921	552,754	65,451	2,763,599	69,459	20.12
1922	581,482	82,070	2,814,941	75,597	18.63
1923	672,617	135,455	2,917,966	82,960	19.23
1924	693,915	86,915	3,039,886	80,919	21.31
1925	704,811	127,210	3,161,810	84,418	20.70
1926	755,827	134,742	3,288,677	88,209	20.42
1927	760,285	131,413	3,405,460	88,573	21.61
1928	816,254	133,016	3,515,442	89,667	21.61
1929	822,198	141,193	3,635,742	92,218	22.10
1930	751,500	119,147	3,702,810	85,803	22.10
1931	703,600	95,211	3,719,694	77,930	19.62
1932	611,800	69,100	3,685,926	68,577	18.43
1933	603,300	63,036	3,642,074	68,744	18.43
1934	668,300	72,261	3,628,941	69,262	18.63
1935	728,300	84,404	3,639,025	73,774	18.93
1936	822,500	108,340	3,695,540	81,575	19.62
1937	865,800	116,353	3,762,608	87,334	18.83
1938	835,600	102,012	3,797,784	80,190	19.62
1939	903,500	120,585	3,862,507	85,803	20.76
1940	980,700	136,303	3,945,521	91,125	23.69
1941	1,148,800	183,970	4,096,776	103,518	25.47
1942	1,360,000	229,151	4,329,638	113,359	27.85
1943	1,583,700	269,298	4,591,813	120,357	29.14
1944	1,714,100	276,264	4,804,742	119,483	30.13
1945	1,693,300	242,855	4,896,667	113,651	31.22
1946	1,505,500	215,735	4,929,967	116,640	30.62
1947	1,495,100	257,875	5,019,781	120,868	30.13
1948	1,560,000	291,303	5,140,316	122,909	30.62
1949	1,550,900	294,899	5,285,005	117,809	32.01
1950	1,686,600	345,504	5,494,417	122,919	33.30
1951	1,815,100	356,476	5,716,491	124,413	33.30
1952	1,887,300	367,447	5,937,394	125,663	34.29
1953	1,973,900	392,164	6,188,547	127,280	37.56
1954	1,960,500	391,654	6,414,843	122,270	39.35
1955	2,099,500	433,277	6,684,991	127,953	38.85
1956	2,141,100	430,935	6,924,888	131,174	38.85

1957	2,183,900	435,866	7,157,984	130,269	39.84
1958	2,162,800	418,833	7,340,662	126,309	41.72
1959	2,319,000	463,953	7,594,864	132,272	43.51
1960	2,376,700	466,357	7,836,872	133,578	43.71
1961	2,432,000	470,568	8,072,548	133,225	44.30
1962	2,578,900	514,763	8,354,421	134,940	47.83
1963	2,690,400	546,897	8,663,731	137,276	49.65
1964	2,846,500	587,743	9,012,907	140,097	51.83
1965	3,028,500	645,272	9,411,797	144,126	54.02
1966	3,227,500	683,714	9,835,779	146,916	57.02
1967	3,308,300	693,556	10,231,152	147,620	58.91
1968	3,466,100	747,120	10,653,728	150,607	62.41
1969	3,571,400	760,064	11,071,613	153,556	65.63
1970	3,578,000	739,764	11,424,775	152,147	67.86
1971	3,697,700	783,096	11,804,436	151,990	69.31
1972	3,898,400	863,164	12,258,669	158,838	72.76
1973	4,123,400	935,651	12,756,051	164,395	75.81
1974	4,099,000	894,004	13,155,176	165,966	74.08
1975	4,084,400	840,255	13,466,597	161,152	72.04
1976	4,311,700	907,749	13,838,519	167,209	76.07
1977	4,511,800	1,007,912	14,290,877	173,140	78.12
1978	4,760,600	1,104,643	14,814,054	180,386	80.12
1979	4,912,100	1,150,502	15,341,921	184,244	81.04
1980	4,900,900	1,080,070	15,752,771	182,185	78.44
1981	5,021,000	1,097,206	16,147,910	184,644	76.57
1982	4,919,300	1,041,772	16,447,136	181,172	73.44
1983	5,132,300	1,138,894	16,824,687	184,208	73.32
1984	5,505,200	1,317,892	17,359,824	194,388	76.97
1985	5,717,100	1,430,937	17,943,972	194,400	76.78
1986	5,912,400	1,498,944	18,534,452	199,432	77.07
1987	6,113,300	1,523,250	19,087,645	204,292	79.63
1988	6,368,400	1,554,809	19,636,149	208,570	83.07
1989	6,591,800	1,618,728	20,167,768	212,477	84.72
1990	6,707,900	1,602,374	20,650,376	214,686	84.34
1991	6,676,400	1,515,484	20,984,075	211,031	84.30
1992	6,880,000	1,606,935	21,348,962	213,049	85.51
1993	7,062,600	1,719,528	21,795,926	217,045	87.30
1994	7,347,700	1,817,471	22,291,432	222,557	89.21
1995	7,543,800	1,916,976	22,829,383	224,681	90.94
1996	7,813,200	2,054,615	23,450,348	227,772	93.93
1997	8,159,500	2,232,978	24,126,422	234,188	94.34
1998	8,508,900	2,469,906	24,908,022	237,465	94.61
1999	8,856,500	2,671,833	25,769,587	240,678	96.87
2000	9,224,000	2,884,434	26,679,930	243,917	98.50

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