OVERFLOW PROBABILITY IN AN ATM QUEUE WITH SELF-SIMILAR INPUT TRAFFIC

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Abstract

Real measurements in high-speed communications networks have recently shown that traffic may demonstrate properties of long-range dependency peculiar to self-similar stochastic processes. Measurements have also shown that, with increasing buffer capacity, the resulting cell loss is not reduced exponentially fast as it is predicted by Markov-model-based queueing theory but, in contrast, decreases very slowly. Presenting a theoretical understanding to those experimental results is still a problem. The paper presents mathematical models for self-similar cell traffic and analyzes the overflow behavior of a finite-size ATM buffer fed by such a traffic. An asymptotical upper bound to the overflow probability, which decreases hyperbollically, h^{-a}, with buffer-size-h is obtained. A lower bound is also described, which demonstrates the same h^{-a} asymptotical behavior, thus showing an actual hyperbolical decay of overflow probability for a self-similar-traffic model.

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1. INTRODUCTION

Recent traffic measurements in corporate LANs, Variable-bit-rate video, ISDN control-channels and other communications systems have indicated traffic behavior of self-similar nature [Leland 94], [Beran]. With actual traffic measurements, it was shown that the overall cell loss decreases very slowly with increasing buffer capacity, in sharp contrast to Poisson-based models where losses decrease exponentially fast with increasing buffer size. The problem is how to get a theoretical explanation of such empirically observed behavior. An extensive bibliographical guide with 420 references to previously published papers on self-similar traffic and analysis is given in [Willinger].

In this paper, four models for self-similar cell input traffic are considered. Asymptotical upper bounds to overflow probability in an ATM buffer queue fed by three types of traffic are presented. These bounds are expressed in terms of the rate of input traffic, capacity of ATM channel, buffer size, and the Hurst parameter (the last shows the level of traffic self-similarity). The derived bounds decrease much slower (hyperbolically) than exponentially with buffer-size growth. For a model with constant source rate 1 and a channel of capacity 1, we are able to compare the obtained upper bounds against the lower bound presented here and proved by us in {TG].

2. MODELS FOR SELF-SIMILAR TRAFFIC

We introduce here mathematical models of self-similar input cell traffic to an ATM buffer.

2. 1. Self-similar processes . First we recall the definition of a self-similar process. Consider a second-order stationary real-number stochastic process $X=(\ldots,X_{-1},X_0,X_1,\ldots)$ of discrete argument (time) $t\in I_{-\infty}=\{\ldots,-1,0,1,\ldots\}$. Denote by $\mu=\mathrm{E}X_t<\infty$ and $\sigma^2=\mathrm{var}\,X_t<\infty$, the mean and the varience of X_t respectively. Denote by *r(k)* the autocorrelation function of process X . The mean μ , the varience σ^2, and the autocorrelation function $r(k)$ do not depend on time t, and $r(k)=r(-k)$.

Now we give the definitions and some properties of exactly and asymptotically self-similar processes (see [Cox], [ST], [Leland 94], [TG]) .

Definition. A process X is called exactly second-order self-similar (e.s.s) with parameter $H=1-\dfrac{\beta}{2},\ 0<\beta<1,$ if its autocorrelation function is

$$r(k)=\frac{1}{2}[(k+1)^{2-\beta}-2k^{2-\beta}+(k-1)^{2-\beta}]\,\hat{=}\,g(k),\quad k\in I_1=\{1,2,\ldots\}.$$

The reason the process X with $r(k)=g(k)$ is called self-similar is that it does not change its correlational structure with averaging. It means that the averaged (over blocks of m) process $(...,X_{-1}^{(m)},X_{0}^{(m)},X_{1}^{(m)},...)$ where $X_t^{(m)}=\frac{1}{m}(X_{tm-m+1}+...+X_{tm}),\quad m\in I_1,\quad t\in I_{-\infty}$ has the autocorrelation function $r^{(m)}(k)$ such that $r^{(m)}(k)=r(k)=g(k).$

H is called the Hurst parameter of process X .

Definition. A process X is called asymptotically second-order self-similar (a.s.o.s.s) with parameter $H=1-\frac{\beta}{2}$, $0<\beta<1,$ if for all $k\in I_1,$

$$\lim_{m\to\infty} r^{(m)}(k)=g(k). \tag{2. 1}$$

The above means that X is a.s.o.s.s if (after averaging over blocks of size m and with $m\to\infty$) its correlational structure tends to that of the e.s.s process.

The important property of X , which guarantees that X is a.s.o.s.s with parameter $H=1-\frac{\beta}{2}$ is $\lim_{k\to\infty}\frac{r(k)}{k^{-\beta}}=c,$ where $0<c<\infty$ is a constant and $0<\beta<1$ [TG]. It is interesting to notice that this limiting equation gives a definition of long-range dependence (l.r.d) process (see [Beran] , Sect. 2. 1) . It means a l.r.d process is always an a.s.o.s.s process.

2. 2. Source process. We consider a traffic as a stream of cells. For convenience, we assume that a cell has the length 1 when it is transmitted over the ATM channel. The channel can transmit C cells at a time unit slot; the time interval $[t,t+1)$ is called slot $t, t\in I_{-\infty}\equiv\{...,-1,0,1,...\}$ and C is called the channel capacity. The case of $C=1$ is considered in Sections 3 and 4; the general case of $C\in I_1$ is considered in Sect. 5.

The discrete-time source process $Y=(...,Y_{-1},Y_0,Y_1,...)$ is a random process where Y_t represents the number of new cells arrived at time t , $Y_t\in I_0\equiv\{0,1,2,...\}$. The source process Y is constructed in the following way.

Let $\theta_s(t-\omega_s+1)$ be a random sequence over time t , associated with a moment of arrival ω_s of source $s,$ $t\in I_{-\infty}$ $,\omega_s\in I_{-\infty}$ $,\theta_s(\ )\in I_0$. The sources are numbered by s according to their arrivals, $\omega_{s+1}\geq\omega_s$. The $\theta_s(1)$ represents the number of cells generated by source s upon its arrival at ω_s and $\theta_s(i)$ gives the analogous number for the moment which is $i-1$ time units later than ω_s . The source s generates its cells within time interval of length $\tau_s\in I_1\equiv\{1,2,3,...\}$ and the sequence $\theta_s(i)$ in this interval $\omega_s\leq i<\omega_s+\tau_s$ is called the source s active period.

We denote by

$$\gamma_s \hat{=} \theta_s(1) + ... + \theta_s(\tau_s) \qquad (2.\ 2)$$

the total number of cells generated by source s during its active period and γ_s is called the volume of source s.

Let ξ_t denote the number of new sources arrived at t. We assume in this paper that ξ_t are independent and Poissonian with parameter λ.

It is assumed that the random sequencies $(\theta_s(1), \ldots, \theta_s(\tau_s))$ are i. i. d. for different s and they are independent of the sequences ξ_t and ω_s.

The source process $Y = (..., Y_{-1}, Y_0, Y_1, ...)$ is defined as

$$Y_t = \sum_s \theta_s(t - \omega_s + 1), \qquad (2.\ 3)$$

i. e. Y is a superposition of source active periods. The Y_t is the total number of cells generated by all active sources at time t.

<u>2. 3. Four models for self-similar traffic.</u> In the next sections, the most detailed results on buffer overflow probability shall be obtained for self-similar input traffics which are the further narrowing of the source process Y. We introduce them now. Let us consider the four special cases of Y.

In the most interesting case of $Y^{(1)}$, $\theta_s(i) = R$ where R is a given constant independent of s and i, $R \in I_1$. The rates R which are less than 1 can not be interpreted in the framework of process $Y^{(1)}$ (the same holds for $Y^{(2)}$ and $Y^{(3)}$), since $\theta_s(i)$ has a meaning of the number of cells arrived at i. However, these rates can be interpreted in the framwork of process $Y^{(4)}$. The process $Y^{(1)}$ with R=1 was considered in [Cox], while $Y^{(1)}$ with arbitrary $R \in I_1$ was considered in [LTG] and [TG].

In the case of $Y^{(2)}$, $\theta_s(i) = R(l)$ given $\tau_s = l$ and the function $R(l)$ is monotonic and not random (given $\tau_s = l$). $R(l)$ does not depend on s and i, $R(l) \in I_1$, $l \in I_1$. However, for the source s without the condition $\tau_s = l$, the rate $R(\tau_s)$ is a random variable.

In the case of $Y^{(3)}$, $\theta_s(i) = R_s(l)$ given $\tau_s = l$ where $R_s(l)$ is a random variable (even for given $\tau_s = l$) and the distribution of $R_s(l)$ does not depend on s and i, $R_s(l) \in I_1$ .According to assumptions in 2.2, the random variables $R_s(l_s)$ (given $\tau_s = l_s$) are independent for different s.

In the case of $Y^{(4)}$, $\theta_s(i), \omega_s \le i < \omega_s + \tau_s$ with different s are the segments of i. i. d. stationary discrete-time processes. The process $Y^{(4)}$ was introduced in [TG]. A condition of asymptotical self-similarity of $Y^{(4)}$ is given by:

Statement 1. If $\boldsymbol{t}^{\alpha} \Pr\{\tau > \boldsymbol{t}\} \to \boldsymbol{c}$, $\boldsymbol{t} \to \infty$, $\mathbf{1} < \alpha < \mathbf{2}$ (where $\boldsymbol{c}$ is a constant, $\mathbf{0} < \boldsymbol{c} < \infty$) and $E\theta < \infty$, $E\theta^2 < \infty$, then $Y^{(4)}$ is asymptotically second-order self-similar process with parameter $\boldsymbol{H} = (\mathbf{3} - \alpha)/\mathbf{2} > \mathbf{0.5}$

In what follows, we give the conditions for self-similarity of processes $Y^{(i)}$, $i = 1,2,3,$ and thus present finally our remaining three self-similar cell traffic models $Y^{(i)}$, $i = 1,2,3$.

Statement 2. The process $Y^{(i)}$ is asymptotically second-order self-similar (a.s.o.s.s) with parameter $H = 1 - \frac{\beta}{2}$, $0 < \beta < 1,$ if asymptotically with $l \to \infty$

$$\left.\begin{aligned} &Y^{(1)}: \quad \Pr\{\tau = l\} \\ &Y^{(2)}: \quad R^2(l)\Pr\{\tau = l\} \\ &Y^{(3)}: \quad (\mathrm{E}R^2(l))\Pr\{\tau = l\} \end{aligned}\right\} \sim c l^{-(\beta+2)} \tag{2. 4}$$

where $c > 0$ is a constant.

For $Y^{(1)}$ with $R = 1$, sufficiency of the condition (2. 4) for a.s.o.s.s was given by Cox [Cox]. Statement 2 for the case of process $Y^{(1)}$ was given in [TG].

3. ATM - BUFFER QUEUE TO CHANNEL OF CAPACITY 1: LOWER BOUND TO OVERFLOW PROBABILITY

In this section, we consider a finite-buffer queueing system with input cell traffic $Y^{(1)}$ having sources of constant rate $\boldsymbol{R} = \mathbf{1}$ and denoted by $Y1$. We will obtain the lower bounds to the stationary buffer-overflow probability, expressed in terms of $\lambda > 0$, $\Pr\{\tau = \boldsymbol{n}\}$ and $\boldsymbol{h}$, where $\boldsymbol{h}$ is the size of the buffer. The bound decays hyperbollically rather then exponentially fast with increasing $\boldsymbol{h}$ when τ has Pareto-type distribution and $Y1$ is self-similar.

Let y_t be the number of new cells arrived at t and z_t be the number of cells that were in the buffer at time t .

Definition. A service discipline $\boldsymbol{d}$ is in class $\boldsymbol{D}(\boldsymbol{h})$, if it satisfies the following two conditions: (i) If $\boldsymbol{y_t} + \boldsymbol{z_t} > \mathbf{0}$, then some cell (out of the $\boldsymbol{y_t} + \boldsymbol{z_t}$ total) goes definitely into service at $\boldsymbol{t}$; (ii) If $\boldsymbol{y_t} + \boldsymbol{z_t} \le \boldsymbol{h} + \mathbf{1}$, then no cells are discarded at time $\boldsymbol{t}$. If $\boldsymbol{y_t} + \boldsymbol{z_t} > \boldsymbol{h} + \mathbf{1}$, instead, then exactly $\boldsymbol{y_t} + \boldsymbol{z_t} - \boldsymbol{h} - \mathbf{1}$ cells are discarded at $\boldsymbol{t}$.

The event { $\boldsymbol{y_t} + \boldsymbol{z_t} > \boldsymbol{h} + \mathbf{1}$} is called the buffer overflow at time $\boldsymbol{t}$. We are interested in the stationary probability of overflow, $\boldsymbol{P_{over}}$.

Our result (the details are in [TG]) for the lower bound to $\boldsymbol{P_{over}}$ is expressed by the following:

Statement 3. The queueing system **Y1/D/1/**$\boldsymbol{h}$ with any service discipline from $\boldsymbol{D}(\boldsymbol{h})$ has the overflow probability $\boldsymbol{P_{over}}$ lowerbounded by

$$P_{over} \ge \frac{1}{(E\tau + E\kappa)^2} \sum_{n=n_1}^{\infty} \Pr\{\tau \ge n\} \tag{3. 1}$$

where $\boldsymbol{n_1} = \left[\frac{\boldsymbol{h}+\boldsymbol{b}}{\boldsymbol{a}}\right] + \mathbf{2},$ $E\kappa$ is given by

$$E\kappa = (1 - e^{-\lambda})^{-1} - 1 \tag{3. 2}$$

and $\boldsymbol{a, b}$ are given by

$$a = \frac{1}{E\tau + E\kappa} \le 1, \qquad b = a + 1 \ge 1.. \tag{3. 3}$$

We apply the bound (3.1) to the most interesting case of Pareto-type distribution, $\Pr\{\tau = \boldsymbol{n}\} = \boldsymbol{c}\boldsymbol{n}^{-\alpha-1}$, $\mathbf{1} < \alpha < \mathbf{2}$, and self-similar traffic **Y1.**

We get

$$P_{over} \ge \frac{c}{\alpha(\alpha-1)(E\tau + E\kappa)^2} \left(\left[\frac{h+b}{a}\right] + 2\right)^{-\alpha+1} \tag{3. 4}$$

where

$$\boldsymbol{E\tau} = \boldsymbol{c}\sum_{\boldsymbol{n}=1}^{\infty} \boldsymbol{n}^{-\alpha}\ , \quad \boldsymbol{c} = \Big(\sum_{\boldsymbol{n}=1}^{\infty} \boldsymbol{n}^{-\alpha-1}\Big)^{-1}, \tag{3. 5}$$

and $\boldsymbol{a}$ and **b** are given by (3. 3) , whereas $E\kappa$ is given by (3.2).

Asymptotically, when $\boldsymbol{h}$ is large, (3.4) gives the bound $P_{over} \ge c\,h^{-\alpha+1} = c\,h^{-\beta} = c\,h^{-2(1-H)}$, where c is a constant independent of $\boldsymbol{h}$ but dependent on λ and α and where we used the equation $H = 1 - \frac{\beta}{2} = \frac{3-\alpha}{2}$ from Statement 1. With this bound, we come to the important conclusion that for the considered self-similar traffic **Y1**, the overflow probability $\boldsymbol{P_{over}}$ cannot decrease faster than hyperbolically with the growth of buffer size $\boldsymbol{h}$.

To get a numerical result, let us consider, for example the case of ***α=1. 5*** (the Hurst parameter is H=(3-α)/2=0. 75) and $\lambda = 0.2$. In this case, **c=0.745, Eτ=1. 95,** ***E*κ=4. 51 ,** and

$$P_{over} \ge \frac{0.0238}{\sqrt{\left[\frac{h+1.15}{0.15}\right]+2}} \tag{3. 6}$$

According to (3. 6) or (3.4),

$$P_{over} \ge 7.9*10^{-3} \quad \text{for } h = 0, \quad P_{over} \ge 2.8*10^{-3}, \quad \text{for } h = 10,$$
$$P_{over} \ge 9.3*10^{-4}, \quad \text{for } h = 100, \ P_{over} \ge 3*10^{-4}, \quad \text{for } h = 1000,$$
$$P_{over} \ge 9.4*10^{-5}, \quad \text{for } h = 10{,}000.$$

4. ATM - BUFFER QUEUE TO CHANNEL OF CAPACITY 1:

UPPER BOUNDS TO OVERFLOW PROBABILITY

Here we consider the processes $Y^{(1)}$ and $Y^{(2)}$ as the input cell traffics to ATM finite-buffer discrete-time queue. We are interested in the buffer-overflow probability. We adopt the usual symbols to denote the queueing systems. **Y/D/1/h/d** means that input traffic is Y (i.e. at time t, the input traffic provides Y_t new requests or, in other words, it provides Y_t new cells), service-time is constant and equal to 1, there is a single server, the buffer has size h, and the sevice discipline is d. In this Section, we restrict our analysis to the case of ATM channel having capacity $C=1$. The cases of greater channel capacity are the subject of the next Section.

We begin with consideration of input traffic $Y^{(1)}$ with $R=1$, denoted by $Y1$.

4. 1. Upper bound to overflow probability for self-similar traffic Y1. A way to get our upper bound is based on the large-deviation results for a sum of independent random variables with long-tailed distribution. In the case of long-tailed distribution, the large deviation of the sum occurs mainly at the expense of just one (maximal) summand. These results originated with Chistyakov [Chistyakov] and Chover et al. [Chover]. The more deep mathematical background for the problem is given in the book of Bingham, Goldie, and Teugles [Bingham] in context of regular variation.

Our upper bound is presented by:

Statement 4. If for the asymptotically self-similar process $Y1$ with parameter $H=\dfrac{3-\alpha}{2}$, the condition $\lambda \mathrm{E}\tau<1$ is satisfied and the source-active-period length τ has the Pareto-type distribution

$$\Pr\{\tau=l\}\sim c_0 l^{-\alpha-1}, \quad 1<\alpha<2, \quad l\to\infty, \tag{4. 1}$$

then for the discrete-time **Y1/D/1/h/d** system with any service discipline $d\in D(h)$, the overflow probability for the size h buffer is upperbounded asymptotically as

$$P_{over}\le \frac{c_0\lambda}{\alpha(\alpha-1)(1-\lambda \mathrm{E}\tau)}h^{-\alpha+1}, \qquad h\to\infty. \tag{4. 2}$$

If for $Y1$ we also consider the other sub-exponential distributions $\Pr\{\tau>l\}$ with long tails (but not only the Pareto-type distribution), for example, the lognormal distribution or the Weibull distribution adopted for the discrete case, and we use the results of Pakes, Cline, Teugels,Willekens (see [Bingham]), then under the conditions of Statement 4 , we get the asymptotical upper bound

$$P_{over}\le \frac{\lambda}{(1-\lambda \mathrm{E}\tau)}\sum_{n=h}^{\infty}\Pr\{\tau\ge n\} . \tag{4. 3}$$

Asymptotically in the case of Pareto-type distribution of τ, the upper bound (4.2) is again hyperbolical over h with the same exponent $(-\alpha+1)$ as in the lower bound (3. 4). This shows that both bounds demonstrate the right asymptotical behavior of P_{over} apart from a factor which is independent of h in (3. 4) and (4.2).

Now we give a numerical example to compare the lower bound (3. 4) and the upper bound (4. 2) for the case of $\Pr\{\tau = l\} = c_0 l^{-\alpha-1}, \quad 1 < \alpha < 2,$ and self-similar traffic $Y1$.

Choosing $\alpha = 1.5, \quad \lambda = 0.2$, we get 35 as the upper bound/lower bound ratio.

4. 2. Upper bounds to overflow probability for self-similar traffics $Y^{(2)}$ and $Y^{(1)}$ with $R > 1$. Here , we extend Statement 4 on the self-similar input traffic $Y^{(2)}$ satisfying (2. 4).

Beginning with the case of $\Pr\{\tau = l\}$ satisfying (4. 1) , we obtain finally

$$P_{over} \le \frac{c_3 \lambda}{\alpha(\alpha-1)(1-\lambda \mathrm{E}\gamma)} h^{-\frac{\alpha+\beta-1}{\alpha-\beta+1}}, \quad h \to \infty \quad , \lambda \mathrm{E}\gamma < 1. \tag{4.4}$$

where $c_3 \equiv c_0 c_2^{-\alpha}$, $c_2 \equiv c_1^{-\frac{2}{\alpha-\beta+1}}$, $c_1 \equiv \sqrt{\frac{c}{c_0}}$,and c is the constant from (2.4).

Thus, (4. 4) is an asymptotical (for large h) upper bound to the overflow probability P_{over} for the discrete-time $\mathbf{Y^{(2)}}$**/D/1/h/d** system with any service discipline $d \in D(h)$ in the case of asymptotically self-similar input cell traffic $Y^{(2)}$ defined in Sec. 2 and satisfying (2. 4) and (4. 1). The bound is expressed in terms of $\lambda, \alpha, \beta,$ and h, where (we remind) λ is the intensity of source arrivals, α is the parameter of distribution of source active-period length τ, $\beta = 2(1-H)$ is the parameter of self-similar traffic ($0 < \beta < 1, \ \beta \le \alpha - 1$), and h is the buffer size.

Since $Y^{(2)}$ is more general process than $Y^{(1)}$, we can derive [from (4. 4)] the upper bound to P_{over} for **YR/D/1/h/d** system where the symbol **YR** denotes the asymptotically self-similar traffic $Y^{(1)}$ with any given rate $R \in I_1$. Putting the restriction $\lambda R \mathrm{E}\tau < 1$, we obtain

$$P_{over} \le \frac{c_0 \lambda R^{\alpha}}{\alpha(\alpha-1)(1-\lambda R \mathrm{E}\tau)} h^{-\alpha+1}, \quad h \to \infty, \quad 1 < \alpha < 2. \tag{4. 5}$$

Thus, (4. 5) is an asymptotical upper bound to the overflow probability P_{over} for discrete-time **YR/D/1/h/d** system.

Next, we consider the case of distribution $\Pr\{\tau = l\}$ which decreases exponentially,

$$\Pr\{\tau = l\} \sim c_4 e^{-\varphi l}, \qquad l \to \infty \tag{4. 6}$$

where $c_4 > 0$ and $\varphi > 0$ are constants.

We obtain,

$$P_{over} \le \frac{c_4 \lambda e^{\varphi}}{\varphi(1-\lambda \mathrm{E}\gamma)} \frac{h^{-1}}{(\ln h)^{\beta}}, \qquad h \to \infty \tag{4. 7}$$

when $\lambda \mathrm{E}\gamma < 1$.

(4. 7) gives an asymptotical upper bound to the overflow probability for a $\mathbf{Y^{(2)}}$**/D/1/h/d** system with self-similar input traffic $Y^{(2)}$ satisfying (2. 4) and (4. 6). The bound (4. 7) obtained for the exponential-tail distribution $\Pr\{\tau = l\}$ goes to zero a little faster than the bounds (4. 2), (4. 4), and (4. 5) obtained for hyperbollical-tail distribution $\Pr\{\tau = l\}$.

5. ATM-BUFFER QUEUE TO CHANNEL OF CAPACITY C : UPPER BOUNDS TO OVERFLOW PROBABILITY

So far we considered an ATM channel which could transmit one cell at its time unit called slot. Now we consider a channel which can transmit C cells in its slot, $C \in I_1$. Again, the problem is to find an upper bound to the buffer-overflow probability.

5. 1. Queueing model . We consider the discrete-time queueing system **Y1/DC/1/h** which has the input cell traffic $Y1$, constant service-time C^{-1} (where C is a positive integer), a single server, and finite buffer of h-cells size. In our consideration , discrete-time **Y1/DC/1/h** is equivalent to the system **Y1/D/C/h** which has C servers, each with constant service-time 1. Taking it into account, it is more convenient for us to analyze **Y1/D/C/h** since, in this case, we should deal with only one channel-scale unit 1 and not partition the channel slots into subslots of length C^{-1} each.

We consider only the service dicsiplines d which satisfy the following two conditions: (i) If $y_t + z_t > 0$ (where y_t and z_t were defined in Sect. 3), $\min(y_t + z_t, C)$ cells go into service at t , (ii) If $y_t + z_t \le h + C$, then no cells are discarded at t . If $y_t + z_t > h + C$, instead, then $y_t + z_t - h - C$ cells are discarded at t .

We denote by $D_C(h)$ the class of the considered disciplines.

The event { $y_t + z_t > h + C$} is called the buffer overflow at time t and

$$P_{over}^t \mathrel{\hat{=}} \Pr\{y_t + z_t > h + C\}$$

denotes its probability. Again, we are interested in getting an upper bound to the steady-state overflow probability denoted by P_{over}, $P_{over} \mathrel{\hat{=}} \limsup_{t\to\infty} P_{over}^t$.

5.2 Upper bound to the overflow probability for self-similar traffic Y1

For the discrete-time **Y1/D/C/h/d,** $d \in D_C(h)$ queueing system with self-similar input traffic $Y1$ satisfying (4. 1), the overflow probability for the size h buffer and channel capacity C is upperbounded by

$$P_{over} \le \frac{c_0 \lambda}{\alpha(\alpha-1)(C-\lambda \mathrm{E}\tau)} h^{-\alpha+1}, \quad h \to \infty, \quad \lambda \mathrm{E}\tau < C. \tag{5. 1}$$

5. 3. Generalization on the input traffic $\mathbf{Y^{(4)}}$. Our consideration in 5. 2 (the details are in [TGa]) remains valid for the self-similar input traffic $Y^{(4)}$ if instead of the distribution of τ, we use the distribution of γ (the source volume),

$$\gamma = \sum_{i=1}^{\tau} \theta(i) \tag{5. 2}$$

where $\theta(i)$ is the generic symbol for $\theta_s(i)$, and if $\Pr\{\gamma = l\}$ is a Pareto-type or sub-exponential distribution. Thus, for $\mathbf{Y^{(4)}}$**/D/C/h/d,** $d \in D_C(h)$, we obtain

$$P_{over} \le \frac{\lambda}{C-\lambda \mathrm{E}\gamma} \sum_{n=h}^{\infty} \Pr\{\gamma > n\}, \quad h \to \infty, \quad \lambda \mathrm{E}\gamma < C. \tag{5. 3}$$

As a first example of $Y^{(4)}$, let us consider the singular case of $\theta(i) = R,\ R \in I_1$. In this case, $Y^{(4)}$ is $Y^{(1)}$. If $\Pr\{\tau = l\}$ satisfies (4. 1), then using $\Pr\{\gamma > x\} = \Pr\{\tau > \frac{x}{R}\}$, we obtain

$$P_{over} \le \frac{c_0 \lambda R^{\alpha}}{\alpha(\alpha-1)(C-\lambda R \mathrm{E}\tau)} h^{-\alpha+1}, \quad h \to \infty, \quad \lambda R \mathrm{E}\tau < \mathrm{C}. \tag{5. 4}$$

The relation (5. 4) is the direct generalization of (4. 5) in the case of channel capacity greater than 1.

As a second example of $Y^{(4)}$, let us consider i. i. d. $\theta(i)$ with

$$\Pr\{\theta(i) = 1\} = 1 - \Pr\{\theta(i) = 0\} = p. \tag{5. 5}$$

If $\Pr\{\tau = l\}$ is the Pareto-type distribution (4. 1), then

$$P_{over} \le \frac{c_0 \lambda p^{\alpha}}{\alpha(\alpha-1)(C-\lambda p \mathrm{E}\tau)} h^{-\alpha+1}, \quad h \to \infty, \quad \lambda p \mathrm{E}\tau < \mathrm{C}. \tag{5. 6}$$

The bound (5. 6) is a natural extention of the bound (4. 5) in the case of $R < 1$ and $C \ge 1$.

These considerations can be extended to the traffic $Y^{(4)}$ with ergodic sequence $\theta(i)$.

5. 4. Generalization on the input traffic $\mathbf{Y^{(2)}}$. The consideration in 5. 2 (the details are in [TGa]) remains also valid for the input traffic $Y^{(2)}$, if instead of the distribution of τ, we use the distribution of $\gamma = \tau R(\tau)$ as in 4. 2. Eventually for the $C \geq 1$ case , if $\lambda \mathrm{E}\gamma < C$, we get the upper bounds which are the same as (4. 4) [for $\mathbf{Y^{(2)}}$**/D/C/**$\mathbf{h/d}$ with the traffic $Y^{(2)}$ satisfying (2. 4) and (4. 1)] and (4. 7) [for $\mathbf{Y^{(2)}}$**/D/C/**$\mathbf{h/d}$ with the traffic $Y^{(2)}$ satisfying (2. 4) and (4. 6)] with the only change of the term $(1-\lambda \mathrm{E}\gamma)$ for the term $(C-\lambda \mathrm{E}\gamma)$ in the denominators.

6. CONCLUSIONS

Four models for ATM cell traffic were considered in this paper. Then a finite buffer fed by self-similar traffic $Y^{(i)}$ was treated as a queueing system. The problem was to find an asymptotical bound to the buffer overflow steady-state probability and with this bound to explain the slow decay of cell-loss probability measured in high-speed networks. Indeed, we got bounds which decrease hyperbollically with increasing buffer size and this decay is much slower than regular exponential decay. The bounds have simple and explicit expressions in terms of intensity and the Hurst parameter of input traffic, ATM channel capacity, and buffer size.

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