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Research Report

Similarity Judgments, Violations of Stationarity, and Reflection Effects in Intertemporal Choice

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Abstract

This paper demonstrates that choices based on similarity judgments, along lines suggested by Rubinstein (1988) and Leland (1994, 1998), will not only exhibit *common ratio* and *reflection* effects under uncertainty but also *common difference* and *reflection* effects in intertemporal contexts.

I. Introduction

Evidence accumulated over many years reveals the inadequacies of the Expected Utility Hypothesis as a descriptive model of choice under uncertainty. Over a much shorter period of time, evidence has accumulated revealing systematic violations of the standard model of choice over time - the Discounted Utility model. In Rubinstein (1988), Azipurrua et al (1993), and Leland (1994, 1998), choice anomalies under uncertainty occur because agents base their decisions on judgments regarding the similarity or dissimilarity of prizes and probabilities across alternatives.[1] This paper specifies conditions under which such a procedure implies analogous violations of the Discounted Utility model in intertemporal settings.

[1] Also see Buchena and Zilberman (1994) and Wilcox and Ballinger (1993).

II. Choice Anomalies Under Uncertainty and Over Time

Axioms assumed in models of choice place restrictions on what agents can choose across different pairs of alternatives. The *independence* axiom, for example, requires that for risky or riskless options L_1, L_2 , and L_3 , if L_1 is weakly preferred to L_2, then the lottery { L_1, p ; L_3, 1-p} must be weakly preferred to the lottery { L_2, p ; L_3, 1-p } for any p. One consequence of this requirement is that preferences between simple lotteries
{ $\$x_1$, p_1 ; \$0, 1-p_1 } and { $\$x_2$, p_2 ; \$0, 1-p_2 } must be invariant to changes in the values of p_1 and p_2 which leave their ratio undisturbed. In choices between S and R and between S' and R' below, for example, independence requires either the choice of S and S' or the choice of R and R'.

S:{ \$3000, .90 ; \$0, .10 }	S':{ \$3000, .02 ; \$0, .98 }
R:{ \$6000, .45 ; \$0, .55 }	R':{ \$6000, .01 ; \$0, .99 }

The *stationarity* assumption of the Discounted Utility model of intertemporal choice places restrictions on how agents can choose between pairs of intertemporal prospects. Consider simple intertemporal prospects T_j and T_k shown below where T_j offers an increment to consumption x_j in time period t_j and T_k offers an increment to consumption x_k in time period t_k.

T_j :	{	x_j	,	t_j	}
T_k :	{	x_k	,	t_k	}

Assuming, for simplicity, a common baseline level of consumption per period, c, agents deciding between these options according to the Discounted Utility model will choose as follows where U(.) is a concave, ratio-scaled, utility function, δ is the one period discount factor and $\succ$ and ~ denote strict preference and indifference, respectively:[2]

1) $T_j \; \overset{\succ}{\underset{\prec}{\sim}} \; T_k$ iff

$$U(c+x_j)\,\delta^{j} + U(c)\,\delta^{k} \;\overset{>}{\underset{<}{=}}\; U(c)\,\delta^{j} + U(c+x_k)\,\delta^{k}$$

Dividing through by δ_j and rearranging terms, yields the following expression:

2) $T_j \; \overset{\succ}{\underset{\prec}{\sim}} \; T_k$ iff

$$U(c+x_j) - U(c) \;\overset{>}{\underset{<}{=}}\; [\,U(c+x_k) - U(c)\,]\,\delta^{k-j}$$

Expression 2 reveals that the only way discounting enters into the decision is through the absolute difference in the time periods. As such, agents given choices between T_1 and T_2 and between T_{11} and T_{12} shown below must either select T_1 and T_{11} or T_2 and T_{12} as the absolute time interval in both choices is identically 1 period.

T_1 :{	$20	,	1 month	}	T_{11} :{	$20	,	11 months	}
T_2:{	$25	,	2 month	}	T_{12}:{	$25	,	12 months	}

Neither the restrictions implied by the independence axiom nor those following from stationarity hold empirically. Instead, regarding the independence axiom, Kahneman and Tversky (1979) among others, find that individuals choosing the safer option S over R, nevertheless choose the riskier option R' over S' as the difference between probabilities declines, their ratio held constant. This phenomenon is referred to as the *common ratio* effect. They also find that for lotteries involving losses, the opposite pattern, RS', obtains. This phenomenon is referred to as the *reflection* effect.

[2] This discussion follows Lowenstein and Prelec (1991a, 1992).

Lowenstein and Prelec (1991, 1992) review evidence revealing parallel failures of the Discounted Utility model. This evidence reveals a tendency for individuals indifferent between two alternatives like T_1 and T_2 to systematically choose the option offering the larger payoff to be received later (e.g. T_{12} over T_{11}) as both are deferred an equal amount into the future. Lowenstein and Prelec refer to this response pattern as the *common difference* effect. In cases where x_j and x_k are future decrements to consumption rather than future increments, the observed response pattern *reflects* in much the same way choices reflect when the options involve uncertain losses rather than gains.

III. Similarity Judgments, the Common Ratio Effect, and Reflection

Building on work by Rubinstein (1988), Leland (1994) proposes a model of choice based on similarity judgments that, given appropriate assumptions, implies common ratio violations of independence and reflection effects. Choice is modeled as a three-step procedure.[3] In the first step, agents attempt to resolve choice by appeal to preference. For alternatives sufficiently different in value, the process terminates here. For alternatives sufficiently close in value, however, agents are assumed to be unable or unwilling to discriminate between the alternatives in terms of preference. In such cases, they next compare prizes and their corresponding probabilities across alternatives in terms of their equality or inequality. In this process, dominant alternatives may be identified, although this need not be the case. If these comparisons fail to reveal a preferable alternative, agents repeat the set of comparisons in terms of the similarity or dissimilarity of prizes and their corresponding probabilities. For this step, let the binary relations $\succ^x$ and $\succ^p$ reading "greater than and dissimilar" be strict partial orders (asymmetric and transitive[4]) on consequences and

[3] For an extended discussion of this model, see Leland (1994).

[4] That is, for all x_g and $x_h \in X$: $x_g >^X x_h \Rightarrow$ not $x_h >^X x_g$, - same for p's $\in$ [0,1]) and for all $x_f > x_g > x_h \in X$: $x_f >^X x_g$, $x_g >^X x_h \Rightarrow x_f >^X x_g$, - same for p's $\in$ [0,1].

probabilities, respectively. As such, the similarity relations, $\sim^x$ and $\sim^p$, defined by $>^x$ and $>^p$ are symmetric[5] but not necessarily transitive in that for some prizes $x_f > x_g > x_h$, $x_f \sim^x x_g$, $x_g \sim^x x_h$ but $x_f >^x x_h$ with the same being possible for probabilities.

For the purposes of choosing between non-dominated prospects like S and R or S' and R', agents first compare the non-zero prizes in the lotteries (i.e., the $3000 and $6000) and their corresponding probabilities (e.g., .90 and .45 in SR) in terms of their similarity or dissimilarity. They then compare the common $0 prizes and their corresponding probabilities (e.g., .10 and .55 in SR) in terms of their similarity / dissimilarity. For each pair of comparisons, agents note whether they "favor" one lottery over the other, are "inconclusive," or are "inconsequential."[6] Once these conclusions have been drawn, agents choose one lottery over the other if it is favored in some comparisons and not disfavored in any (i.e., the remaining conclusion is either inconclusive or inconsequential), and at random otherwise.

To see how this procedure works, as well as what configurations of prize and probability similarities and dissimilarities are viewed as favoring one lottery over another, being inconclusive, or being inconsequential, consider the choice between S and R reproduced below.

S:{	$3000,	.90	;	$0,	.10	}
R:{	$6000,	.45	;	$0,	.55	}

Given these alternatives, agents will first compare $6000 with $3000 and .90 with .45. If $6000 $>^x$ $3000 and .90 $\sim^p$ .45 this paired comparison will favor R as it offers a noticeably

[5] That is, for all x_g and $x_h \in X$: $x_g \sim^X x_h \Rightarrow x_h \sim^X x_g$ - same for p's $\in [0,1]$.

[6] In what follows, whether comparisons "favor" one lottery over the other, are "inconclusive," or are "inconsequential" will be obvious. For a more general discussion, see Leland (1994).

better prize at similar probability. If , $\$6000 \sim^{x} \$3000$ and $.90 >^{p} .45$, the paired comparison will favor S at it offers a similar good outcome at noticeably better odds. For $\$6000 >^{x} \$3000$ and $.90 >^{p} .45$, the paired comparison is inconclusive as R offers a noticeably better prize but S offers a good outcome at noticeably higher probability. Finally, if $\$6000 \sim^{x} \$3000$ and $.90 \sim^{p} .45$, the paired comparison is inconsequential as the options offer similar outcomes at similar probabilities.

Agents next compare the $0 outcome with itself and .55 with .1. If probabilities appear dissimilar, the pair of comparisons will favor the safe option as it offers a noticeably lower probability of the worst possible outcome. If the probabilities are similar, the paired comparison is deemed inconsequential – both lotteries offer the same worst outcome at similar probability.

Given the two paired comparisons, the following eight configurations of similarity and dissimilarity perceptions and associated conclusions regarding which lottery to choose are possible. Agents with perceptions corresponding to 3a will choose R to the extent that the first paired comparison favors this option (R offers a better prize at similar probability) and the second is inconsequential. This is the only similarity configuration favoring the risky option. Agents with perceptions corresponding to 3b – 3d will choose at random to the extent that both paired comparisons are inconsequential (3b), make contradictory recommendations (3c), or are inconclusive (3d). Agents with the any of the four remaining similarity/dissimilarity perceptions will choose S because it offers: a similar good outcome at dissimilar and greater probability, the same worst outcome at dissimilar and lower probability, or both

3a) $\$6000 >^{x} \$3000$	$.90 \sim^{p} .45$	;	$\$0 \sim^{x} \$0$	$.55 \sim^{p} .10$	
favors R			inconsequential		Choose R

3b) $\$6000 \sim^{x} \$3000$ $.90 \sim^{p} .45$; $\$0 \sim^{x} \$0$ $.55 \sim^{p} .10$

inconsequential inconsequential Choose at random

3c) $\$6000 >^{x} \$3000$ $.90 \sim^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

favors R favors S Choose at random

3d) $\$6000 >^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 \sim^{p} .10$

inconclusive inconsequential Choose at random

3e) $\$6000 \sim^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

favors S favors S Choose S

3f) $\$6000 \sim^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 \sim^{p} .10$

favors S inconsequential Choose S

3g) $\$6000 >^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

inconclusive favors S Choose S

3h) $\$6000 \sim^{x} \$3000$ $.90 \sim^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

inconsequential favors S Choose S

Now consider choices between S' and R' reproduced below:

S':{	\$3000,	.02	;	\$0,	.98	}
R':{	\$6000,	.01	;	\$0,	.99	}

For individuals choosing between S and R as described above to adhere to the independence axiom given the choice between S' and R', their similarity perceptions on probabilities must remain unaltered as values of the probabilities are reduced, their ratio held constant. As shown in Rubinstein (1988), this requires that the similarity relation $\sim^p$ be a λ-ratio similarity such that $p_1 \sim^p p_2$ if $1/\lambda \leq p_1/p_2 \leq \lambda$, $\lambda>1$. Suppose instead that $\sim^p$ is an ε-difference similarity such that $p_1 \sim^p p_2$ if $| p_1 - p_2 | \leq \varepsilon$. Suppose further than the reductions in the values of the probabilities from SR to S'R' (.90 and .45 to .02 and .01) are sufficient to result in the latter probabilities (and their complements) appearing similar (i.e., $.02 \sim^p .01$ and $.99 \sim^p .98$.) If so, then agents choosing S in SR will either choose at random in S'R' (for 3e,f and h) or choose R (for 3g). The reason is that all those paired comparisons favoring S in 3e though 3h will be inconclusive in S'R' if the probabilities are perceived as similar. Those agents choosing at random between S and R because the paired comparisons were in conflict (3c) or both inconclusive (3d), will choose R in S'R'.

Those agents choosing at random in RS because both paired comparisons were inconsequential (3b) will also choose at random in R'S' since the reduction in probabilities has no impact on agents' perceptions regarding their similarity. Likewise agents choosing R (3a) will also choose R'. These results are summarized in 3a' – 3h' below where perceptions changed as a result of the reduction in probabilities are shown in bold.

3a') $\$6000 >^x \$3000$ $.02 \sim^p .01$; $\$0 \sim^x \$0$ $.99 \sim^p .98$

favors R — inconsequential — Choose R

3b') $\$6000 \sim^x \$3000$ $.02 \sim^p .01$; $\$0 \sim^x \$0$ $.99 \sim^p .98$

inconsequential — inconsequential — Choose at random

3c') $\$6000 >^x \$3000$ $.02 \sim^p .01$; **$\$0 \sim^x \$0$ $.99 \sim^p .98$**

favors R — **inconsequential** — **Choose R**

3d') **$\$6000 >^x \$3000$ $.02 \sim^p .01$** ; $\$0 \sim^x \$0$ $.99 \sim^p .98$

favors R — inconsequential — **Choose R**

3e') **$\$6000 \sim^x \$3000$ $.02 \sim^p .01$** ; **$\$0 \sim^x \$0$ $.99 \sim^p .98$**

inconsequential — **inconsequential** — **Choose at random**

3f') **$\$6000 \sim^x \$3000$ $.02 \sim^p .01$** ; $\$0 \sim^x \$0$ $.99 \sim^p .98$

inconsequential — inconsequential — **Choose at random**

3g') $\$6000 >^x \$3000$ $.02 \sim^p .01$; **$\$0 \sim^x \$0$ $.99 \sim^p .98$**

favors R — **inconsequential** — **Choose R**

3h') $\$6000 \sim^x \$3000$ $.02 \sim^p .01$; **$\$0 \sim^x \$0$ $.99 \sim^p .98$**

inconsequential — **inconsequential** — **Choose at random**

For choices between lotteries SR and S'R' involving losses, this analysis implies that if $\sim^p$ is an ε-difference similarity, agents' choices between losses will be the reflection of those between gains.[7] For example, given a choice between S:{-$3000,.90; $0,.10} and R:{-$6000, .45; $0,.55}, R will be selected to the extent that while the first paired comparison is inconclusive (-$3000 $\not\sim^x$ -$6000 but .90 $\not\sim^p$ .45) the second favors R as it offers a noticeably greater probability (.55 $\not\sim^p$ .10) of the best possible outcome, $0. If S' and R' involve losses, on the other hand, S' will be selected to the extent that it offers a similar probability (.02 $\sim^p$ .01) of a noticeably better, albeit unfortunate, outcome (-3000>^x$ -$6000) and a similar probability (.99 $\sim^p$ .98) of the best possible outcome, $0.

III. Similarity, Common Difference Effects, and Reflection

The preceding discussion reveals that a simple model of similarity judgments will account for common ratio and reflection effects observed in choice under uncertainty.[8] Now consider how such a decision process might also account for the *common difference* and *reflection* effects observed in intertemporal choice. For this purpose consider again a choice between simple intertemporal prospects T_1 and T_2.

T_1 :{ $20 , 1 month }
T_2:{ $25 , 2 month }

[7] Note that simple reflection of preferences when the gains in a pair of lotteries are replaced with losses occurs even if $\sim^p$ is an λ-difference similarity.

[8] More generally, Leland (1994, 1998) shows that this model of similarity judgments implies behaviors implied by Kahneman and Tversky's (1979) Prospect theory for risky alternatives represented as prospects and behaviors implied by Loomes and Sugden's (1982) Regret theory for alternatives represented in state-matrices used to test predictions of that theory. In addition, similarity predicts violations of transitivity, dominance and invariance not predicted by expected utility or proposed alternatives.

As per the procedure assumed under uncertainty, agents given these choices will compare consumption increments \$20 and \$25, and their dates of receipt, 1 month and 2 months, in terms of their similarity or dissimilarity. They then conclude whether the pair of comparisons favors one alternative over the other, is inconclusive, or is inconsequential. How these conclusions are drawn, in turn, depends on how agents feel about more immediate versus delayed consumption. Assume, consistent with the assumption of impatience, that sooner receipts are preferred to later receipts, ceteris paribus. If so, then any of the following four configurations of similarity/dissimilarity perceptions associated with the choice between T_1 and T_2 are possible where $>^t$ and $\sim^t$ are the time analogs to the dissimilarity and similarity relations on prizes and probabilities:

4a) $25 \sim^x 20$, $2 \sim^t 1$

inconsequential Choose at random

4b) $25 >^x 20$, $2 \sim^t 1$

favors T2 Choose T2

4c) $25 \sim^x 20$, $2 >^t 1$

favors T1 Choose T1

4d) $25 >^x 20$, $2 >^t 1$

inconclusive Choose at random

Agents with perceptions as in 4a will choose at random to the extent that consumption increments and their dates of receipt are perceived as similar (i.e., the pair of comparisons is inconsequential). Agents perceiving 25 as greater than and dissimilar to 20 but 2 months as similar to 1 month will choose the larger-later option T_2 as it offers a noticeably better prize at a similar date in the future (i.e., the pair of comparisons favors T_2) . In 4c, agents choose the smaller-sooner alternative T_1 to the extent that it offers a similar consumption increment noticeably sooner (i.e., the pair of comparisons favors T_1). Finally, agents with perceptions as in 4d will choose at random since T_2 offers a noticeably better consumption increment but T_1 offers a desirable increment at a noticeably earlier date (i.e., the pair of comparisons is inconclusive.)

Now suppose that agents are given a choice between prospects T_{11} and T_{12} shown below:

T_{11} :{ $20 , 11 month }

T_{12} :{ $25 , 12 month }

As noted earlier, the Stationarity property of the Discounted Utility model requires that preference between intertemporal prospects be invariant to manipulations that defer changes in consumption an equal distance into the future. In the current context, this requires that agents choose either T_1 and T_{11} or T_2 and T_{12}. For agents with perceptions conforming to 4 a, b, c, or d, to adhere to this requirement, the similarity relation $\sim^t$ must be an ε-difference similarity such that $t_j \sim^t t_k$ if $|t_j - t_k| \leq \varepsilon$ (e.g., agents perceiving 2 $\sim^t$ 1 will also perceive 12 $\sim^t$ 11). Suppose instead that dates differing by the same absolute amount appear more similar the further they are deferred into the future as would be the case if $\sim^t$ was a λ-difference similarity such that $t_j \sim^t t_k$ if $1/\lambda \leq t_j/t_k \leq \lambda$, $\lambda>1$. If so, then as the dates associated with consumption increments are deferred into the future, individuals choosing at random between T_1 and T_2 (case 4a) and those choosing the later-larger option T_2 (case 4b) will continue to do so. Those choosing the smaller-sooner option T_1 because it appears to offer a similar payoff noticeably sooner (4c) will switch to choosing at random for sufficient delays into the future. Finally, those choosing randomly as in 4d will eventually switch to the choice of later-larger option like T_{12} (their perceptions change to those in 4b). This result constitutes the *common-difference* effect.

If we replace consumption increments in choices between T_1 and T_2 and between T_{11} and T_{12} with consumption decrements, reflection effects also follow and do so even if $\sim^t$ is an ε-difference similarity (i.e., even if agents' choices for increments are consistent with stationarity). As an example, consider an individual choosing T_1 over T_2 because increments to future consumption of $25 and $20 appear similar but receipt in 2 months is noticeably inferior to receipt in 1 month. If the choices instead involved either paying $20 in 1 month or paying $25 in 2 months then, to the extent that -$25 and -$20 appear similar but payment in 2 months is noticeably superior to receipt in 1 month, the option offering the larger payment in a later period will be recommended by similarity judgments.

IV. Discussion

The descriptive adequacy of the axioms and assumptions of Expected Utility was questioned early on by Friedman and Savage (1948), Markowitz (1952), and Allais (1953) and later by Kahneman and Tversky (1979) and a cast of thousands. A plethora of models of choice under uncertainty abandoning or relaxing one or more of the offended axioms of expected utility have been proposed. Much more recently, the axioms and assumptions of the Discounted Utility model have come under scrutiny and models abandoning stationarity have followed.

This paper has examined the conditions under which a small subset of observed violations of Expected and Discounted Utility will occur if agents based choices, at least in part, on similarity judgments. As shown, similarity judgments account for *common ratio*, *common difference*, and *reflection* effects. These findings, in turn, suggest that the "solution" to anomalies problems, be they associated with choice under uncertainty or over time, may not lie in the axioms and assumptions of preference based models of decision making. Instead, the solution may lie in a more detailed understanding of the actual process whereby decisions are made. Further inquiry into this possibility seems in order.

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